Some Matrix Basics

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A matrix is a rectangular array of numbers.

$$\begin{bmatrix} 43 & 2 \\ 18 & 7 \\ 6 & 42 \end{bmatrix}$$

We say the dimensions of this matrix are 3 x 2 because it has 3 rows and 2 columns.

A matrix with the same number of rows as columns is called a square matrix.

A matrix with only one column is called a vector.

$$\begin{bmatrix} 8 \\ 3 \\ -1 \\ 7 \end{bmatrix}$$

The transpose of a matrix is obtained by exchanging the rows and columns. The transpose of a matrix M is denoted M′.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}' = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

Matrix Addition

$$\begin{bmatrix} 2 & 9 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ -8 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 15 \\ -7 & 3 \end{bmatrix}$$

Defined for matrices with identical dimensions.

Matrix Multiplication

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix} = \begin{bmatrix} 1\times7+2\times9 & 1\times8+2\times10 \\ 3\times7+4\times9 & 3\times8+4\times10 \\ 5\times7+6\times9 & 5\times8+6\times10 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 28 \\ 57 & 64 \\ 89 & 100 \end{bmatrix}$$

Number of columns of the first matrix must match the number of rows of the second matrix.

Multiplication of a matrix by a single number:

$$5 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5\times1 & 5\times2 \\ 5\times3 & 5\times4 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$$

The Identity Matrix

A square matrix with ones on the main diagonal and zeros elsewhere is called the identity matrix. Such a matrix is usually denoted I. The 3 x 3 identity matrix is

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

AI=A and IB=B for any matrices A and B of appropriate dimensions.

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The Inverse of a Matrix

If A is a square matrix and there exists a matrix B such that AB=I, then B is called the inverse of the matrix A. Usually we denote the inverse of a matrix A by A^{-1}.

$$\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0.50 & 0.00 \\ -0.25 & 0.50 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus, $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.50 & 0.00 \\ -0.25 & 0.50 \end{bmatrix}$

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Some Matrix Operations in R

- t(A) gives the transpose of a matrix A.
- A+B computes the sum of the matrices A and B.
- A%*%B computes AB, the product of the matrices A and B.
- solve(A) gives the inverse of a square matrix A if the inverse exists.
- det(A) computes the determinant of a square matrix A.

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