STAT 611 **Fall 2012** **HW 2**

Due: Tuesday, September 11

1. Prove that for any matrix $\boldsymbol{A}$ of rank r, there exist nonsingular matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$ such that
$$\boldsymbol{PAQ} = \begin{bmatrix} \underset{r\times r}{\boldsymbol{I}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}.$$

2. Prove the following: If $rank(\underset{m\times n}{\boldsymbol{A}}) = n$ and $rank(\underset{k\times l}{\boldsymbol{C}}) = k$, then $rank(\boldsymbol{ABC}) = rank(\boldsymbol{B})$.

3. Prove that if $\underset{m\times n}{\boldsymbol{A}}$ has full column rank, then $\boldsymbol{A}'\boldsymbol{A}$ is nonsingular.

4. Complete exercises 22 through 25 from Appendix A.9.

5. Read definition A.11 on pages 260 and 261 of the text. Suppose $\boldsymbol{A}$ is an arbitrary matrix. Prove the uniqueness of the Moore-Penrose generalized inverse of $\boldsymbol{A}$.