The LOWESS Smoother

1/19/2011

LOWESS

"lowess" stands for

LOcally WEighted polynomial regreSSion.

The original reference for lowess is

Cleveland, W. S. (1979). Robust locally weighted regression and smoothing scatterplots. JASA **74** 829-836.

How is the lowess curve determined?

Suppose we have data points (x_1,y_1), (x_2,y_2),...(x_n,y_n).

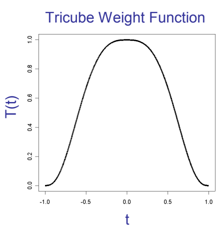
Let $0 < f \le 1$ denote a fraction that will determine the smoothness of the curve.

Let r = n*f rounded to the nearest integer.

Consider the tricube weight function defined as

$T(t) = (1 - |t|^3)^3$ for $|t| < 1$

$= 0$ for $|t| \ge 1$.

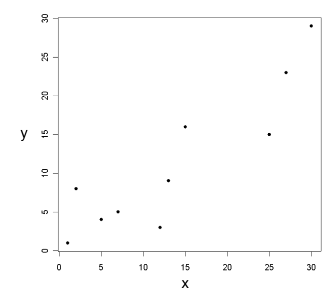


For i=1, ..., n; let h_i be the r^{th} smallest number among $|x_i-x_1|$, $|x_i-x_2|$, ..., $|x_i-x_n|$.

For k=1, 2, ..., n; let $w_k(x_i)=T((x_k - x_i) / h_i)$.

An Example

i	1	2	3	4	5	6	7	8	9	10
x_i	1	2	5	7	12	13	15	25	27	30
y_i	1	8	4	5	3	9	16	15	23	29



Suppose a lowess curve will be fit to this data with f=0.4.

Table Containing $|x_i-x_j|$ Values

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
x_1	0	1	4	6	11	12	14	24	26	29
x_2	1	0	3	5	10	11	13	23	25	28
x_3	4	3	0	2	7	8	10	20	22	25
x_4	6	5	2	0	5	6	8	18	20	23
x_5	11	10	7	5	0	1	3	13	15	18
x_6	12	11	8	6	1	0	2	12	14	17
x_7	14	13	10	8	3	2	0	10	12	15
x_8	24	23	20	18	13	12	10	0	2	5
x_9	26	25	22	20	15	14	12	2	0	3
x_{10}	29	28	25	23	18	17	15	5	3	0

Calculation of h_i from $|x_i-x_j|$ Values

n=10, f=0.4 ➔ r=4

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	
x_1	0	1	4	6	11	12	14	24	26	29	h_1= 6
x_2	1	0	3	5	10	11	13	23	25	28	h_2= 5
x_3	4	3	0	2	7	8	10	20	22	25	h_3= 4
x_4	6	5	2	0	5	6	8	18	20	23	h_4= 5
x_5	11	10	7	5	0	1	3	13	15	18	h_5= 5
x_6	12	11	8	6	1	0	2	12	14	17	h_6= 6
x_7	14	13	10	8	3	2	0	10	12	15	h_7= 8
x_8	24	23	20	18	13	12	10	0	2	5	h_8=10
x_9	26	25	22	20	15	14	12	2	0	3	h_9=12
x_{10}	29	28	25	23	18	17	15	5	3	0	h_{10}=15

Weights $w_k(x_i)$ Rounded to Nearest 0.001

k

i	1	2	3	4	5	6	7	8	9	10
1	1.000	0.986	0.348	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.976	1.000	0.482	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.193	1.000	0.670	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.820	1.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	1.000	0.976	0.482	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.986	1.000	0.893	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.850	0.954	1.000	0.000	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.976	0.670
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.986	1.000	0.954
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.893	0.976	1.000

$w_6(x_5) = (1 - (| x_6 - x_5 | / h_5)^3)^3 = (1 - (| (13 – 12) / 5 |)^3)^3 = (1 – 1 / 125)^3 \approx 0.976$

Next consider local weighted regressions

For each i=1, 2, ..., n; let $\hat{\beta}_0^*(x_i)$ and $\hat{\beta}_1^*(x_i)$

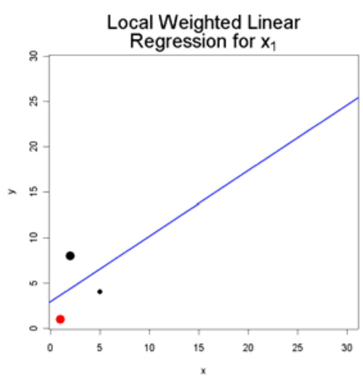
denote the values of β_0 and β_1

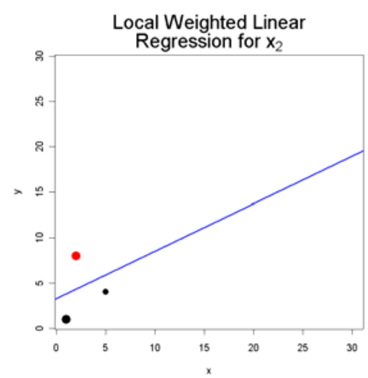
that minimize $\sum_{k=1}^{n} w_k(x_i)(y_k - \beta_0 - \beta_1 x_k)^2$.

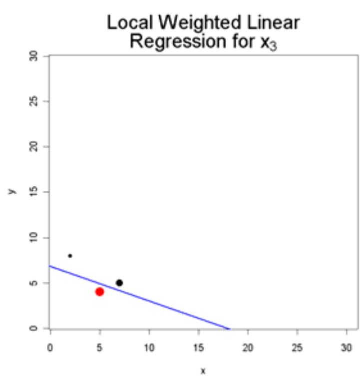
For i=1, 2, ..., n; let $\hat{y}_i^* = \hat{\beta}_0^*(x_i) + \hat{\beta}_1^*(x_i)x_i$

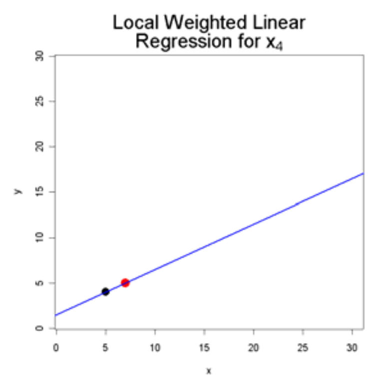
and

$e_i = y_i - \hat{y}_i^*$

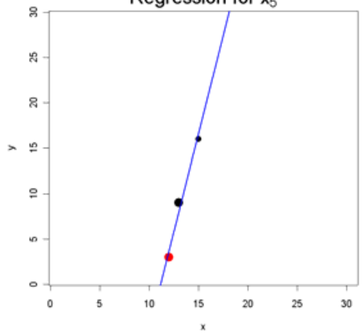








Local Weighted Linear Regression for x_5



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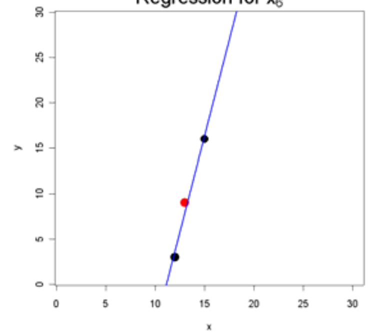
Local Weighted Linear Regression for x_6



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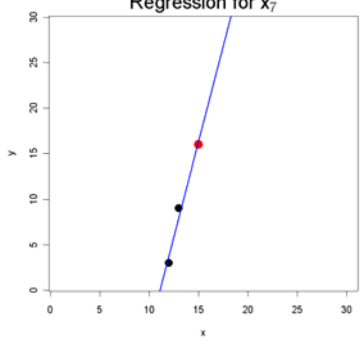
Local Weighted Linear Regression for x_7



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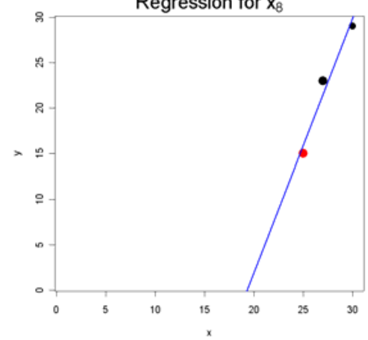
Local Weighted Linear Regression for x_8



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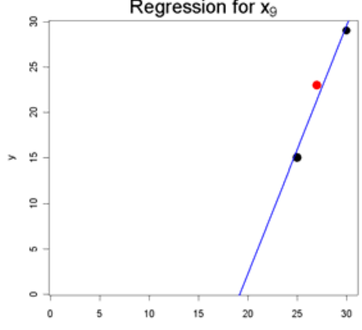
Local Weighted Linear Regression for x_9



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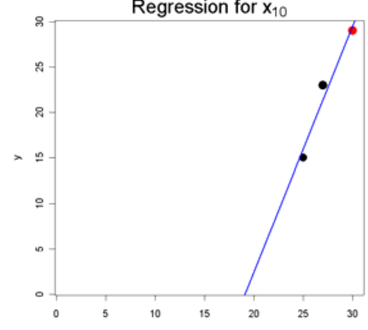
Local Weighted Linear Regression for x_{10}



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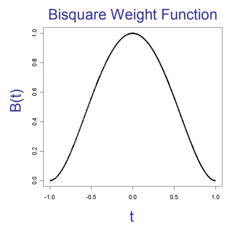
Next measure the degree to which an observation is outlying.

Consider the bisquare weight function defined as

$B(t) = (1 - t^2)^2$ for $|t| < 1$

$= 0$ for $|t| \geq 1$.



For k=1,2,...,n; let

$$\delta_k = B(e_k/(6s))$$

where s is the median of $|e_1|, |e_2|, ..., |e_n|$.

Now down-weight outlying observations and repeat the local weighted regressions.

For each i=1, 2, ..., n; let $\hat{\beta}_0(x_i)$ and $\hat{\beta}_1(x_i)$

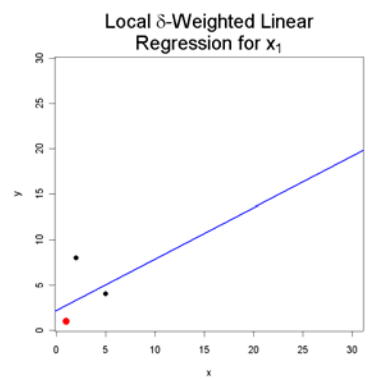
denote the values of β_0 and β_1

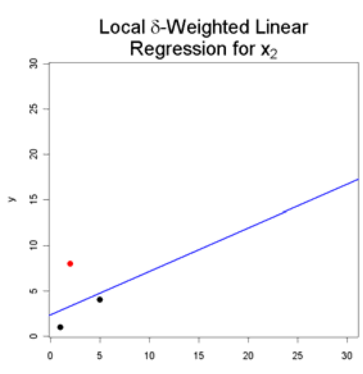
that minimize $\sum_{k=1}^{n} \delta_k w_k(x_i)(y_k - \beta_0 - \beta_1 x_k)^2$.

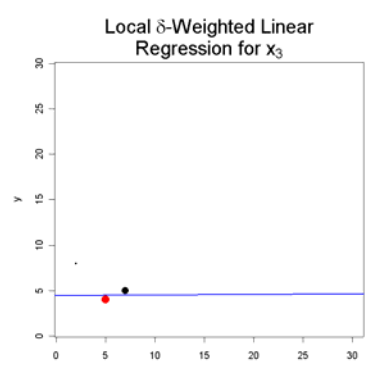
For i=1, 2, ..., n; let $\hat{y}_i = \hat{\beta}_0(x_i) + \hat{\beta}_1(x_i)x_i$.

Iterate one more time.

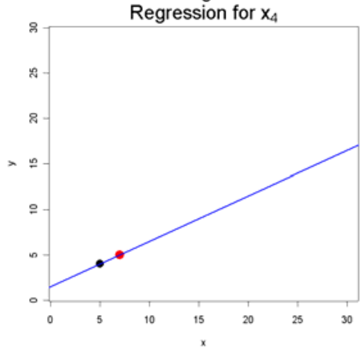
Now use the new fitted $\hat{y}_i$ values to compute new δ_k as described previously. Substitute the new δ_k for the old δ_k and repeat the local weighted regressions one last time to obtain the final $\hat{y}_i$ values. These resulting $\hat{y}_i$ values are the lowess fitted values. Plot these values versus $x_1, x_2, ..., x_n$ and connect with straight lines to obtain the lowess curve.



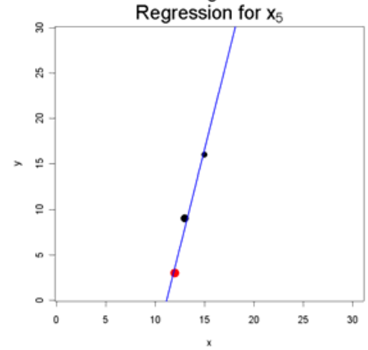




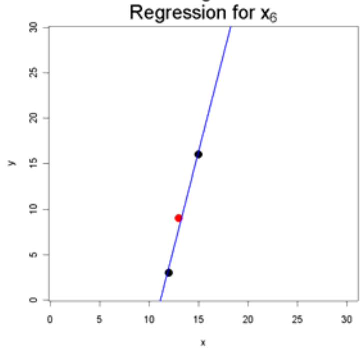
Local δ-Weighted Linear Regression for x_4



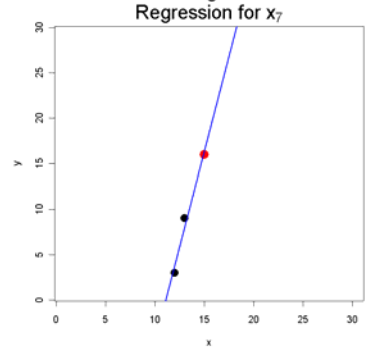
Local δ-Weighted Linear Regression for x_5



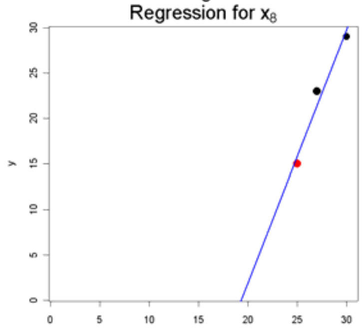
Local δ-Weighted Linear Regression for x_6



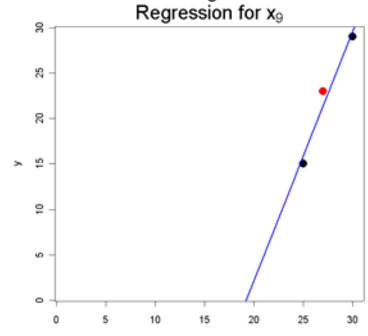
Local δ-Weighted Linear Regression for x_7



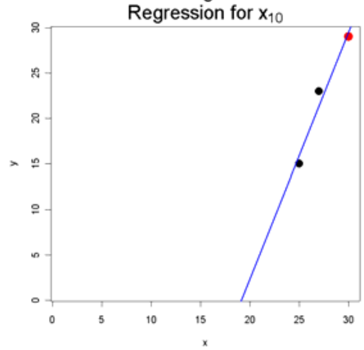
Local δ-Weighted Linear Regression for x_8



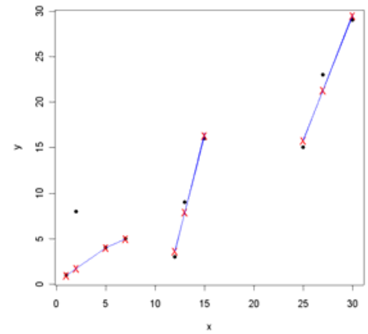
Local δ-Weighted Linear Regression for x_9



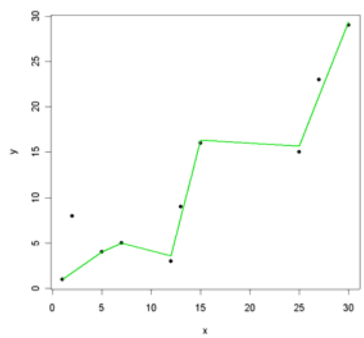
Local δ-Weighted Linear Regression for x_{10}



Plot Showing All 10 Lines and Predicted Values after One More Iteration



The Lowess Curve



lowess in R

```
o=lowess(x,y,f=0.4)
plot(x,y)
lines(o$x,o$y,col=2,lwd=2)
```

o$x will be a vector containing the x values.

o$y will contain the lowess fitted values for the values in o$x.

f controls the fraction of the data used to obtain each fitted value.


```
o=lowess(y~x,f=.2)
plot(fossil)
lines(o,lwd=2)

#See also the function 'loess' which has more
#capabilities than 'lowess'.
```