Analysis of a Split-Split-Plot Experiment

An experiment was conducted to measure the effect of three factors,

$$A = \text{row spacing}, \qquad B = \text{plant density}, \qquad \text{and} \qquad C = \text{date},$$

on a complex response variable y. The response variable is related to the relationship between a plant's leaf area and the amount of light intercepted at various levels in its canopy. Destruction of several plants is necessary to obtain a single value of y.

A field was divided into $r = 4$ blocks. Each block was divided into two whole plots, and $a = 2$ row spacings (38 and 76 cm) were randomly assigned to the whole plots within each block. Each whole plot was divided into four split plots, and $b = 4$ plant densities were randomly assigned to the split plots within each whole plot. Each split plot was divided into eight split-split plots, and $c = 8$ dates were randomly assigned to each split-split plot. At each of the eight dates during the growing season, the appropriate split-split plots were used to obtain $rab = (4)(2)(4) = 32$ measures of the response variable. A grand total of $rabc = (4)(2)(4)(8) = 256$ measurements are available for analysis. We can consider the following model for the data

$$\begin{aligned} y_{ijkl} &= \mu + \rho_l + \alpha_i + (wp)_{il} && \text{(whole-plot portion)} \\ &\quad +\beta_j + (\alpha\beta)_{ij} + (sp)_{ijl} && \text{(split-plot portion)} \\ &\quad +\gamma_k + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + (ssp)_{ijkl}, && \text{(split-split-plot portion)} \\ &\quad (i = 1, \ldots, a \quad j = 1, \ldots, b \quad k = 1, \ldots, c \quad l = 1, \ldots, r) \end{aligned}$$

where $(wp)_{il} \sim N(0, \sigma^2_{wp})$, $(sp)_{ijl} \sim N(0, \sigma^2_{sp})$, $(ssp)_{ijkl} \sim N(0, \sigma^2_{ssp})$, and all random effects are independent. We may partition as follows:

Whole Plot Partitioning

SOURCE	DF	DF
Block	$r-1$	3
A	$a-1$	1
W.P. Error	$(r-1)(a-1)$	3
C. Total(wp)	$ra-1$	7

Split Plot Partitioning

SOURCE	DF	DF
Whole Plot	$ra-1$	7
B	$b-1$	3
AB	$(a-1)(b-1)$	3
S.P. Error	$(r-1)a(b-1)$	18
C. Total(sp)	$rab-1$	31

Split-Split Plot Partitioning

SOURCE	DF	DF
Split Plot	$rab-1$	31
C	$c-1$	7
AC	$(a-1)(c-1)$	7
BC	$(b-1)(c-1)$	21
ABC	$(a-1)(b-1)(c-1)$	21
S.S.P. Error	$(r-1)ab(c-1)$	168
C. Total(ssp)	$rabc-1$	255

W.P. Error is Block$*A$.

S.P. Error is Block$*B$+Block$*AB$.

S.S.P. Error is Block$*C$+Block$*AC$+Block$*BC$+Block$*ABC$.

SAS code for the analysis of the data described on the front of this handout is provided below.

```
proc mixed method=type3 data=one;
  class rep spacing density date;
  model y=spacing
          density spacing*density
          date spacing*date density*date spacing*density*date / ddfm=satterthwaite;
  random rep rep*spacing rep*spacing*density;
run;
```

The mean squares computed by SAS (after some rounding) are as follows:

Source	Mean Square
rep	0.0469
spacing	0.0283
rep*spacing	0.0554
density	0.0132
spacing*density	0.1004
rep*spacing*density	0.0395
date	3.6455
spacing*date	0.0815
density*date	0.0253
spacing*density*date	0.0240
Residual	0.0389

Test for the significance of main effects and interactions among the fixed factors. In this case the error term for testing for differences among the levels of a fixed factor will be the random term associated with the experimental units to which the levels of the factor were randomly assigned. The error term for testing for significant interaction will be the error term with the most degrees of freedom among the error terms for the factors in the interaction.