Interpretation of the Slope of the Least-Squares Regression Line

If we regress Y against X to get the least-squares regression equation $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$, we can interpret the slope $\hat{\beta}_1$ as follows:

If $\hat{\beta}_1 > 0$, we could say something like, "An increase of one unit in X is associated with an estimated increase of $\hat{\beta}_1$ units in the mean of Y."

If $\hat{\beta}_1 < 0$, we could say something like, "An increase of one unit in X is associated with an estimated decrease of $-\hat{\beta}_1$ units in the mean of Y."

If we regress $\log(Y)$ against X to get the least-squares regression equation $\widehat{\log(Y)} = \hat{\beta}_0 + \hat{\beta}_1 X$, we can interpret the slope $\hat{\beta}_1$ as follows:

"An increase of one unit in X is associated with an estimated multiplicative change of $e^{\hat{\beta}_1}$ in the median of Y."

Note that if $\hat{\beta}_1 > 0$, then the multiplicative factor will be greater than 1, suggesting that the median of Y increases with increasing X.

On the other hand if $\hat{\beta}_1 < 0$, then the multiplicative factor will be less than 1, suggesting that the median of Y decreases with increasing X.

If we regress Y against $\log(X)$ to get the least-squares regression equation $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 \log(X)$, we can interpret the slope $\hat{\beta}_1$ as follows:

If $\hat{\beta}_1 > 0$, we could say something like,"An increase by a multiplicative factor of 2 in X is associated with an estimated increase of $\hat{\beta}_1 \log(2)$ units in the mean of Y."

If $\hat{\beta}_1 < 0$, we could say something like, "An increase by a multiplicative factor of 2 in X is associated with an estimated of decrease of $-\hat{\beta}_1 \log(2)$ units in the mean of Y."

If we regress $\log(Y)$ against $\log(X)$ to get the least-squares regression equation $\widehat{\log(Y)} = \hat{\beta}_0 + \hat{\beta}_1 \log(X)$, we can interpret the slope $\hat{\beta}_1$ as follows:

"An increase by a multiplicative factor of 2 in X is associated with an estimated multiplicative change of $2^{\hat{\beta}_1}$ in the median of Y."

Note that if $\hat{\beta}_1 > 0$, then the multiplicative factor will be greater than 1, suggesting that the median of Y increases with increasing X.

On the other hand if $\hat{\beta}_1 < 0$, then the multiplicative factor will be less than 1, suggesting that the median of Y decreases with increasing X.

If a multiplicative factor is between 1 and 2, it is often more clear to describe changes in terms of a percent increase. A multiplicative factor of $1.X$ corresponds to an X% increase. For example, a multiplicative factor of 1.42 corresponds to a 42% increase.

If a multiplicative factor is between 0 and 1, it is often more clear to describe changes in terms of a percent decrease. A multiplicative factor of $0.X$ corresponds to an $100 - X$% decrease. For example, a multiplicative factor of 0.77 corresponds to a 23% decrease.