A Contrast that Tests for a Linear Trend

Researchers wished to investigate the relationship between the amount of fertilizer and yield in a certain field. A portion of the field was divided into 28 plots. Four fertilizer treatments (10, 20, 30, and 40 units) were assigned to the 28 plots in a completely randomized design. A table summarizing the yield data for the 28 plots is provided below.

Amount of Fertilizer Applied to Plot	Number of Plots Treated	Average Yield per Plot (bushels)
10	7	21
20	7	27
30	7	32
40	7	36

1. Suppose the pooled estimate of standard deviation was $s_p = 10.0$. Test

   $$H_0 : \mu_{10} = \mu_{20} = \mu_{30} = \mu_{40} \text{ vs. } H_A : \mu_{10}, \mu_{20}, \mu_{30}, \mu_{40} \text{ are not all equal.}$$

   Provide a test statistic, approximate p-value, and a conclusion.

2. Sketch a plot of the average yields. Put *Amount of Fertilizer* on the horizontal axis and *Average Yield* on the vertical axis.

3. When group means are associated with quantitative levels of an explanatory variable (e.g., amount of fertilizer), we may wish to test for a linear trend. Formally we test

   $$H_0 : \mu_{10} = \mu_{20} = \mu_{30} = \mu_{40} \text{ vs. } H_A : \mu_x = \beta_0 + \beta_1 x \text{ for } x = 10, 20, 30, 40; \text{ some } \beta_0; \text{ and some } \beta_1 \neq 0.$$

   The alternative just says that the points $(10, \mu_{10}), (20, \mu_{20}), (30, \mu_{30})$, and $(40, \mu_{40})$ fall along a straight line with nonzero slope. This is equivalent to testing whether a certain contrast of the means is 0. The coefficients of the contrast are given by $C_x = x - \bar{x}$. Provide a test statistic, approximate p-value, and a conclusion.