

Tests and Confidence Intervals for Linear Combinations of Means

To examine the effect of pets in stressful situations, researchers recruited 45 women with dogs as pets. Fifteen of the subjects were randomly assigned to each of three groups. Each woman in group 1 was asked to do a stressful task alone. Each woman in group 2 was asked to do the same stressful task with a good friend present. Each woman in group 3 was asked to do the same stressful task with her dog present. The mean heart rate during the task was the measure of the effect of stress. Higher mean heart rates are considered an indication of greater stress. Although 15 women were randomly assigned to each group, one woman in the friend group and one woman in the pet group dropped out of the study. A partially completed ANOVA table and some summary statistics are provided below.

Source	D.F.	Sum of Squares	Mean Squares	F	P-Value
Between or Model		2320.1			< 0.0001
Within or Error					
Total		5839.1			

Group	Description	Mean Heart Rate
1	Alone	82.524
2	Friend	91.242
3	Pet	73.042

- Fill in the missing entries in the ANOVA table.
- Are there any significant differences among the group means? Write down null and alternative hypotheses, give a test statistic, p -value, and conclusion in nontechnical terms.

There are many other questions that the researchers might want to answer. Is there a significant difference between the mean of the pet group and mean of each of the other two groups? Is there a significant difference between the mean heart rate of the pet group and the average of the means for the other two groups? What is an appropriate interval estimate of the mean heart rate for those in the pet group? All of these questions can be answered by forming appropriate linear combinations of the observed sample averages and computing the proper standard errors.

A linear combination of the sample averages may be written as

$$g = C_1\bar{Y}_1 + C_2\bar{Y}_2 + \cdots + C_I\bar{Y}_I.$$

For example, the difference between the sample average for the friend group and the pet group is obtained by choosing $C_1 = 0$, $C_2 = 1$, and $C_3 = -1$.

$$g = C_1\bar{Y}_1 + C_2\bar{Y}_2 + C_3\bar{Y}_3 = (0)\bar{Y}_1 + (1)\bar{Y}_2 + (-1)\bar{Y}_3 = \bar{Y}_2 - \bar{Y}_3 = 91.242 - 73.042 = 18.200.$$

Under the one-way ANOVA model,

$$\text{Mean}(g) = \gamma = C_1\mu_1 + C_2\mu_2 + \cdots + C_I\mu_I, \quad \text{SE}(g) = s_p \sqrt{\frac{C_1^2}{n_1} + \frac{C_2^2}{n_2} + \cdots + \frac{C_I^2}{n_I}}, \quad \text{and } t = \frac{g - \gamma}{\text{SE}(g)}$$

has a t -distribution with $n - I$ degrees of freedom.

